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# Analysis of Cantor's Continuum Hypothesis

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**Abstract**

In this paper, the entire infinite set of positive rational numbers is presented. This set is corresponded with the infinite set of all natural numbers, divided into groups; these groups consist of the same number of natural numbers. Initially, the infinite set of rational numbers is linearly ordered in two mutually perpendicular directions. In this paper, a radically new approach to the continuum hypothesis is developed (taking into account the conclusions of K. Gödel and P. Cohen). The main content of this work is concentrated in six tables. The author's main argumentation is presented in tables 5 and 6, as well as in Section 4.

## 1. Introduction

**1.1.1** We believe that it is most appropriate to give the following quotation as an introduction to this work.

Paul J. Cohen. "Set Theory and The Continuum Hypothesis", 1966 [1] (a monograph by Paul. J. Cohen, Chapter III, p. 85)

> "Our goal is to present Gödel's proof that if ZF is consistent then it remains consistent if the Generalized Continuum Hypothesis (GCH) and AC are added. The Continuum Hypothesis (CH) was first stated by Cantor in 1878. It was listed by Hilbert first in his list of unsolved problems given in his famous address of 1900. Despite many attempts the problem remained unsolved."

**1.1.2** It is necessary to point out Kurt Gödel's attempt to prove Cantor's continuum hypothesis. In his proof published in 1940 [2], Gödel concluded that in the axiom system ZFC (the axiom system of the Zermelo-Fraenkel theory with the axiom of choice) the continuum hypothesis cannot be disproved. Cohen proved in 1963 that in the ZFC axiom system, Cantor's continuum hypothesis can neither be disproved nor proved.

## 2. Approach of Cantor

**2.1.1** Let us present one more quote — Cantor's letter to Dedekind dated November 29, 1873 (see [3], p. 12–13).

> "Gestatten Sie mir, Ihnen eine Frage vorzulegen, die für mich ein gewisses theoretisches Interesse hat, die ich mir aber nicht beantworten kann; vielleicht können Sie es, und sind so gut, mir darüber zu schreiben, es handelt sich um folgendes.
>
> Man nehme den Inbegriff aller positiven ganzzahligen Individuen $n$ und bezeichne ihn mit $(n)$; ferner denke man sich etwa den Inbegriff aller positiven reellen Zahlgrößen $x$ und bezeichne ihn mit $(x)$; so ist die Frage einfach die, ob sich $(n)$ dem $(x)$ so zuordnen lasse, dass zu jedem Individuum des einen Inbegriffes ein und nur eines des andern gehört? Auf den ersten Anblick sagt man sich, nein es ist nicht möglich, denn $(n)$ besteht aus diskreten Teilen, $(x)$ aber bildet ein Kontinuum; nur ist mit diesem Einwande aber nichts gewonnen und so sehr ich mich auch zu der Ansicht neige, dass $(n)$ und $(x)$ keine eindeutige Zuordnung gestatten, so kann ich doch den Grund nicht finden und um den ist es mir zu tun, vielleicht ist es ein sehr einfacher.
>
> Wäre man nicht auch auf den ersten Anblick geneigt zu behaupten, dass sich $(n)$ nicht eindeutig zuordnen lasse dem Inbegriffe $(\frac{p}{q})$ aller positiven rationalen Zahlen $\frac{p}{q}$? Und dennoch ist es nicht schwer zu zeigen, dass sich $(n)$ nicht nur diesem Inbegriffe, sondern noch dem allgemeineren
>
> $$(a_{n_1,n_2,\ldots,n_\nu})$$
>
> eindeutig zuordnen lässt, wo $n_1, n_2, \ldots, n_\nu$ unbeschränkte positive ganzzahlige Indizes in beliebiger Zahl $\nu$ sind."

Let us translate:

> "Allow me to put a question to you that has a certain theoretical interest for me, but which I cannot answer; maybe you can, and are good enough to write to me about it, it's about the

following.
Let us take the totality [Inbegriff] of all positive integer individuals $n$ and denote it by $(n)$; Next, we consider the totality of all positive real numbers $x$ and denote it with $(x)$; so the question is simply whether $(n)$ can be assigned to $(x)$ in such a way that each individual of the one totality has one and only one of the other? At first sight one says to oneself, no it is not possible, because $(n)$ consists of discrete parts, but $(x)$ forms a continuum; but nothing is gained with this objection and no matter how much I incline to the opinion that $(n)$ and $(x)$ do not allow a one-to-one correlation, I still cannot find the reason and that is what I am concerned with, maybe it's a very simple one.
*Wouldn't one be inclined to claim at first sight that (n) cannot be uniquely assigned to the totality* $(\frac{p}{q})$ *of all positive rational numbers* $\frac{p}{q}$*? And yet it is not difficult to show that (n) belong not only to this totality, but also to the more general one*

$$(a_{n_1,n_2,\ldots,n_\nu})$$

*can be uniquely assigned, where* $n_1, n_2, \ldots, n_\nu$ *are unbounded positive integer indices in any number* $\nu$*."*

(We added italics to this quote — in the last part of the text.)

**2.1.2** This letter shows the following.

1. Cantor, back in 1873, was looking for an answer to the question of the countability (or uncountability) of "*the set of all real positive numerical quantities*", i.e. **continuum of real positive numbers**.
2. As is well known, Cantor began research in the field of set theory in 1872. And already by November 1873 he proved (***as he was sure***) that it is possible to correspond "the totality of all positive individuals $n$" [**i.e. the set of all natural numbers**] with "the totality $(\frac{p}{q})$ of all positive rational numbers $\frac{p}{q}$" so, "that each individual [*number*] of the one totality has one and only one [*number*] of the other".
(We added the words in square brackets to the text of the quotation)
3. It follows from this: **Cantor was sure that he could prove that the set of all rational numbers is a countable set.**
4. This subsequently led Cantor to the **continuum hypothesis**, i.e. to the conclusion that between the set of **all natural numbers** and the set (***continuum***) **of all real numbers** ***there is no uncountable infinite number set***.

**2.2.1** Let us examine the ordered **set of *all* positive rational numbers**. We present Table 1 (see Appendix), in which each letter denotes a rational number, for example: $a_{1,1}, a_{1,2}, a_{1,3}, a_{1,4}, \ldots$; while the first index (in this example — $1, 1, 1, 1, \ldots$) corresponds to the numerator of a certain fraction, and the second index $(1, 2, 3, 4, \ldots)$ corresponds to the denominator of the same fraction. (*Table 1 — unlike all other tables in this paper, is nominal, abstract.*)

**2.2.2** By using ***natural numbers*** let us count **rational numbers** (the sequence of counting rational numbers is indicated in the table by arrows). ***According to Cantor*, we *can* count** in this way — *moving along the arrows* — ***all*** **rational numbers** that make up *the infinite set under study.*

**2.2.3** Let us move on to Table 2 (see Appendix). When constructing this table, the following restriction is introduced: it *contains only such rational numbers*, the numerator and denominator of which is **one of the following numbers**: $1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.

**2.2.4** Table 2 is a certain specific example. It relates to what is stated above in §2.2.1 and §2.2.2.

**We state:** *by moving along the arrows,* — from number to number — *we* ***can*** **count** using natural numbers *all the rational numbers given in Table 2* (in square brackets, under each *rational number*, the natural number corresponding to it — according to the account — *is indicated*).

# 3. The set of all rational numbers

**3.1.1** Let us once again ***count*** **all positive rational numbers**. At the same time, ***according to Cantor's approach*** (as expressed in the quote given in Section 2, §2.1.1), we *must correspond* ***all*** *positive* **integers (i.e. natural numbers)** ***with all*** *positive* **rational numbers** *so that each* **integer (natural) number** *number uniquely matches to one and only one* ***rational*** **number**.

**3.1.2** Let us study *the set of* ***all*** *positive* **rational numbers**, *linearly ordered in two mutually perpendicular directions.* A certain fragment of the specified ordered infinite set is given in Tables 3 and 4 (see Appendix).

**3.1.3** As Tables 3 and 4 show, one direction of the linear ordering of rational numbers ("horizontally") is associated with the denominator of the fraction, and the second ("vertical") is associated with the numerator of the fraction: "vertically" the numbers **increase**, and "horizontally" they are decreasing. These tables also include those rational numbers that have already been discussed earlier — based on Table 2.

**3.1.4** When constructing Tables 3 and 4, a condition similar to the condition (restriction) that is introduced when constructing Table 2 is adopted — *only such rational numbers* are presented whose numerator and denominator can be written **as one of the following numbers**: $1, 2, 3, 4, \ldots 17, 18, 19, 20$. At the top of Tables 3 and 4 are **natural numbers**.

**3.2.1** Let us examine **natural numbers**. ***Let us transform*** *the linearly ordered* **set of *all* natural numbers**: we ***divide*** this infinite set — starting from 1 and continuing to $\infty$ — ***into identical finite groups of numbers***.

**3.2.2** Let there be 10 numbers in each group. Consequently, a linearly ordered **set of all natural numbers** can be represented as follows: ***1st group, 2nd group, 3rd group, 4th group,...***

**3.2.3** Let us introduce Table 5 (see appendix). Table 5 — *generally* — displays the ***entire*** **ordered set of *all* positive rational numbers** (this *generalized* table also includes repeating rational numbers; these numbers are not marked).

**3.2.4** **Table 5 presents 900 rational numbers** (these numbers are ordered as indicated in §3.1.3 when describing Tables 3 and 4):

- in the "**part A**" of 100 numbers, 37 numbers are repeating;
- in "**part B**" of 300 numbers, 108 numbers are repeating;
- in the "**part C**" of 500 numbers, 200 numbers are repeating

(respectively, 63 numbers, 192 numbers and 300 numbers are not repeating).

**3.3.1 We state.**

1. Parts of Table 5 (A, B, C) differ from each other by 200 numbers.
2. "Part B" and "part C" are shaped like "*right angles*".

**3.3.2 Table 5 can be continued *unlimitedly***:

- first construct the "**part D**" (in the shape of a "right angle"), which will have 700 numbers; 276 of them are repeating (and, accordingly, 424 are non-repeating, additional numbers);
- then construct the "**part E**" (of the same shape) which will have 900 numbers; 332 of them are repeating (and, accordingly, 568 are not repeating, additional numbers);
  **and so on**.

**3.3.3** Let us move on to Table 6. This table shows the "***group 93***" (the group of natural numbers from 921 to 930) and **the part of the infinite set of all positive rational numbers** corresponding to the "***group 93***".

**3.3.4** Above, in §3.3.2, it is indicated that **Table 5 can be continued unlimitedly**. The portion of the infinite set of positive rational numbers presented in Table 6 is an example (*arbitrarily chosen*) of a continuation of Table 5.

**3.4.1** For each of the parts (A, B, C) of Table 5 described in §3.2.4, *one or another group of* 10 *natural numbers is indicated*: group 1, group 2 and group 3.

**3.4.2** From this table it clearly follows that it is **impossible** — ***impossible in principle*** — to match the two infinite sets under study so that one natural number would correspond to one (and only one) rational number, two natural numbers would correspond to two (and only two) rational numbers, three natural numbers would correspond to numbers would correspond to three (and only three) rational numbers...(**see what is stated in §2.1.1 and §3.1.1**).

## 4. Conclusions

### Part I

**4.1.1** As is well known, Cantor counted *rational numbers* moving along the lines indicated in Tables 1, 2, 3, 4. Such **counting** leads to a **logical error**: *it creates the confidence* that — sequentially moving from one rational number to another (see Table 4) — **we can count (enumerate) *all the numbers*** that make up the infinite set of *positive rational numbers*.

**4.1.2** Relatively recently, at the end of the 20th century, N. Galkin and G. Wilf proposed another **way to count** *positive rational numbers* [4] that make up this infinite set. This **method of counting** also leads to the above **logical error**.

**4.2.1** The infinite sets of rational and natural numbers must be represented so that they are ***directly ("straight") corresponded with one another***.

**4.2.2** This problem was solved by constructing a special table — Table 5, which can be **continued unlimitedly** (see §3.3.4). Let us examine the first part of the table — Part A; Table 5 shows:

- **to count (enumerate) all *rational numbers*** of part A, natural numbers from 1 to 41 were taken (see Table 3), **however, only *natural numbers* $\mathbf{1, 2, 3, \ldots 8, 9, 10}$ can be used**;
- *natural numbers* $11, 12, 13, \ldots 18, 19, 20$ cannot be used in part A, because they are necessary to count (enumerate) all the numbers in part B;
- you also cannot use the *natural numbers* $21, 22, 23, \ldots 28, 29, 30$, because they are needed to count (enumerate) all the numbers in Part C;
- *Natural numbers* $31, 32, 33, \ldots 38, 39, 40$, cannot be used in part A since they are necessary to count (enumerate) all the numbers in part D;
  and so on.

**4.2.3** This means that the 63 *rational numbers* of **part A should be counted** using the 10 *natural numbers* of group 1 — counted so that one natural number corresponds to one (and only one) rational number, two natural numbers correspond to two (and only two) rational numbers, and so on.

**4.2.4 Further similar.**

**Part B**: required to count 192 *rational numbers* using only **10** *natural numbers* of group 2.
**Part C**: required to count **300** *rational numbers* using only 10 *natural numbers* of group 3.
**Part D**: required to count 424 *rational numbers* using only 10 *natural numbers* of group 4.
And so on.

**4.2.5 We state** (conclusion):

***the set of all positive rational numbers is an uncountable set.***

## Part II

**4.3.1 Let us present the second proof of the conclusion stated above.**

**4.3.2** Let us point out the following:

- the approach proposed by Cantor allows us to sequentially **count** — *one after another* — rational numbers (see tables 1, 2, 3, 4);
- but Cantor faced a different, entirely different, task — **to count (enumerate) *all*** *rational numbers.*

**4.4.1** Let us examine table 5 once again. It is constructed as follows:

$$\begin{array}{cccccccccccc} 1, & 2, & 3, & 4, \ldots & 11, & 12, & 13, & 14, \ldots & 21, & 22, & 23, & 24, \ldots \\ 1/1, & 1/2, & 1/3, & 1/4, \ldots & 1/11, & 1/12, & 1/13, & 1/14, \ldots & 1/21, & 1/22, & 1/23, & 1/24, \ldots \end{array}$$

Here:

*The top row* — is the sequence of all natural numbers (the natural numbers are written above the table) — this is **the entire infinite set** of natural numbers.

*The bottom row* — is the first row in the table of all (positive) rational numbers; this row **is a small part** of the mentioned infinite set.

**4.4.2** Let us establish a ***one-to-one correspondence*** between the corresponding elements of the two rows:

1 and 1/1, 2 and 1/2, 3 and 1/3, 4 and 1/4, …and so on.

From such a correspondence, it follows that to count (enumerate) all rational numbers that are in the first (infinite) row of table 5, we must use (“spend”) ***the entire*** *infinite set of natural numbers.* **But then, what will we use to count (enumerate) all the remaining rational numbers?!**

# Appendix

## Transfinite ordinals

**1.1** Let us adopt the following initial assumption (basic condition).

If given:

$$\infty + 1, +2, +3, \ldots,$$

then

$$\infty + 1 = \infty, \quad \infty + 2 = \infty, \quad \infty + 3 = \infty, \ldots$$

**1.2** Therefore, in accordance with §1.1

$$\infty + 1, 2, 3, \ldots = \infty$$

**2.1** Consider 2 ordered sets of all natural numbers (we will denote them by $\bar{N}_1$ and $\bar{N}_2$), written sequentially

$$1, 2, 3, \ldots\ldots\ldots\ldots, \infty; \qquad 1, 2, 3, \ldots\ldots\ldots\ldots, \infty$$

**2.2** We state:

> **ordered sets $\bar{N}_1$ and $\bar{N}_2$ are 2 identical, autonomous and unrelated sets of all natural numbers** (*for the opposite statement, see §4.1*).

**3.1** Consider 4 ordered sets of all natural numbers: $\bar{N}_1, \bar{N}_2, \bar{N}_3, \bar{N}_4$, written sequentially in the form of a "straight line"

$$1, 2, 3, \ldots\ldots, \infty; \qquad 1, 2, 3, \ldots\ldots, \infty; \qquad 1, 2, 3, \ldots\ldots, \infty; \qquad 1, 2, 3, \ldots\ldots, \infty$$

**3.2** We state:

> **there are 4 ordered sets of all natural numbers — identical, autonomous and not connected with each other** (*for the opposite statement, see §4.1*).

**4.1** As is well-known, Cantor (based on the concept of ***"actual infinity"*** that he introduced) considered the sequences of sets of natural numbers presented in §2.1 and §3.1 *as a single whole, constructed from transfinite ordinals.*

**4.2 But this is unacceptable, incorrect: $\infty$ *does not have an end* — *"the end of infinity".*** If we assume that the sets presented in §2.1 and §3.1 constitute "*one whole*", then for them there will be

$$1, 2, 3, \ldots\ldots\ldots\ldots, \underbrace{\infty; \qquad 1, 2, 3}_{\infty+1,2,3,\ldots}, \ldots\ldots\ldots\ldots, \infty;$$

$$1, 2, 3, \ldots\ldots, \underbrace{\infty; \qquad 1, 2, 3}_{\infty+1,2,3,\ldots}, \ldots\ldots, \underbrace{\infty; \qquad 1, 2, 3}_{\infty+1,2,3,\ldots}, \ldots\ldots, \underbrace{\infty; \qquad 1, 2, 3}_{\infty+1,2,3,\ldots}, \ldots\ldots, \infty$$

In accordance with §1.2

$$\infty + 1, 2, 3, \ldots = \infty$$

*This means that all transfinite ordinals will be radically changed —* ***"completely destroyed"***.

**5.1** Consider 4 ordered sets of all positive rational numbers; we will denote them by $\bar{R}_1, \bar{R}_2, \bar{R}_3, \bar{R}_4$.

**5.2 $\bar{R}_1, \bar{R}_2, \bar{R}_3, \bar{R}_4$ are 4 identical, autonomous and unrelated to each other ordered sets of all rational numbers.**

**6.1** Let us correspond — **pairwise** — 4 ordered sets of all natural numbers (described in §2.1–§3.2) and 4 ordered sets of all positive rational numbers

> $\bar{N}_1 \quad \bar{R}_1$, $\bar{N}_2 \quad \bar{R}_2$, $\bar{N}_3 \quad \bar{R}_3$, $\bar{N}_4 \quad \bar{R}_4$

**6.2** *What was shown in our article for infinite sets $\bar{N}_1$ and $\bar{R}_1$ is also true for infinite sets $\bar{N}_2 \quad \bar{R}_2$, $\bar{N}_3 \quad \bar{R}_3$, $\bar{N}_4 \quad \bar{R}_4$.*

## References


[1] Paul J. Cohen. *Set theory and the continum hypothesis*. New York, W.A. Benjamin, 1966, p. 163.

[2] Kurt Gödel. *The consistency of the continuum hypothesis*. Princeton University Press, 1940.

[3] Georg Cantor. *Briefwechsel Cantor-Dedekind*. German. Ed. by E. Noether and J. Cavaillès. Paris: Hermann, 1937, p. 62.

[4] H. Wilf and N. Calkin. "Recounting the rationals". In: *The American Mathematical Monthly* 107.4 (Apr. 2000), pp. 360–363.

# Appendix

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $a_{1,1}$ | $a_{1,2}$ | $a_{1,3}$ | $a_{1,4}$ | $a_{1,5}$ | $a_{1,6}$ | $a_{1,7}$ | $a_{1,8}$ | $a_{1,9}$ | $a_{1,10}$ | ... |
| $a_{2,1}$ | $a_{2,2}$ | $a_{2,3}$ | $a_{2,4}$ | $a_{2,5}$ | $a_{2,6}$ | $a_{2,7}$ | $a_{2,8}$ | $a_{2,9}$ | $a_{2,10}$ | ... |
| $a_{3,1}$ | $a_{3,2}$ | $a_{3,3}$ | $a_{3,4}$ | $a_{3,5}$ | $a_{3,6}$ | $a_{3,7}$ | $a_{3,8}$ | $a_{3,9}$ | $a_{3,10}$ | ... |
| $a_{4,1}$ | $a_{4,2}$ | $a_{4,3}$ | $a_{4,4}$ | $a_{4,5}$ | $a_{4,6}$ | $a_{4,7}$ | $a_{4,8}$ | $a_{4,9}$ | $a_{4,10}$ | ... |
| $a_{5,1}$ | $a_{5,2}$ | $a_{5,3}$ | $a_{5,4}$ | $a_{5,5}$ | $a_{5,6}$ | $a_{5,7}$ | $a_{5,8}$ | $a_{5,9}$ | $a_{5,10}$ | ... |
| $a_{6,1}$ | $a_{6,2}$ | $a_{6,3}$ | $a_{6,4}$ | $a_{6,5}$ | $a_{6,6}$ | $a_{6,7}$ | $a_{6,8}$ | $a_{6,9}$ | $a_{6,10}$ | ... |
| $a_{7,1}$ | $a_{7,2}$ | $a_{7,3}$ | $a_{7,4}$ | $a_{7,5}$ | $a_{7,6}$ | $a_{7,7}$ | $a_{7,8}$ | $a_{7,9}$ | $a_{7,10}$ | ... |
| $a_{8,1}$ | $a_{8,2}$ | $a_{8,3}$ | $a_{8,4}$ | $a_{8,5}$ | $a_{8,6}$ | $a_{8,7}$ | $a_{8,8}$ | $a_{8,9}$ | $a_{8,10}$ | ... |
| $a_{9,1}$ | $a_{9,2}$ | $a_{9,3}$ | $a_{9,4}$ | $a_{9,5}$ | $a_{9,6}$ | $a_{9,7}$ | $a_{9,8}$ | $a_{9,9}$ | $a_{9,10}$ | ... |
| $a_{10,1}$ | $a_{10,2}$ | $a_{10,3}$ | $a_{10,4}$ | $a_{10,5}$ | $a_{10,6}$ | $a_{10,7}$ | $a_{10,8}$ | $a_{10,9}$ | $a_{10,10}$ | ... |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... |

Table 1

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 1/1 [1] | 1/2 [2] | 1/3 [5] | 1/4 [6] | 1/5 [11] | 1/6 [12] | 1/7 [21] | 1/8 [22] | 1/9 [31] | 1/10 [32] |
| 2/1 [3] | | 2/3 [7] | | 2/5 [13] | | 2/7 [23] | | 2/9 [33] | |
| 3/1 [4] | 3/2 [8] | | 3/4 [14] | 3/5 [20] | | 3/7 [30] | 3/8 [34] | | 3/10 |
| 4/1 [9] | | 4/3 [15] | | 4/5 [24] | | 4/7 [35] | | 4/9 | |
| 5/1 [10] | 5/2 [16] | 5/3 [19] | 5/4 [25] | | 5/6 [36] | 5/7 | 5/8 | 5/9 | |
| 6/1 [17] | | | | 6/5 [37] | | 6/7 | | | |
| 7/1 [18] | 7/2 [26] | 7/3 [29] | 7/4 [38] | 7/5 | 7/6 | | 7/8 | 7/9 | 7/10 |
| 8/1 [27] | | 8/3 [39] | | 8/5 | | 8/7 | | 8/9 | |
| 9/1 [28] | 9/2 [40] | | 9/4 | 9/5 | | 9/7 | 9/8 | | 9/10 |
| 10/1 [41] | | 10/3 | | | | 10/7 | | 10/9 | |

Table 2

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | .... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1/1 [1] | 1/2 [2] | 1/3 [5] | 1/4 [6] | 1/5 [11] | 1/6 [12] | 1/7 [21] | 1/8 [22] | 1/9 [31] | 1/10 [32] | 1/11 | 1/12 | 1/13 | 1/14 | 1/15 | 1/16 | 1/17 | 1/18 | 1/19 | 1/20 | .... |
| 2/1 [3] | | 2/3 [7] | | 2/5 [13] | | 2/7 [23] | | 2/9 [33] | | 2/11 | | 2/13 | | 2/15 | | 2/17 | | 2/19 | | .... |
| 3/1 [4] | 3/2 [8] | | 3/4 [14] | 3/5 [20] | | 3/7 [30] | 3/8 [34] | | 3/10 | 3/11 | | 3/13 | 3/14 | | 3/16 | 3/17 | | 3/19 | 3/20 | .... |
| 4/1 [9] | | 4/3 [15] | | 4/5 [24] | | 4/7 [35] | | 4/9 | | 4/11 | | 4/13 | | 4/15 | | 4/17 | | 4/19 | | .... |
| 5/1 [10] | 5/2 [16] | 5/3 [19] | 5/4 [25] | | 5/6 [36] | 5/7 | 5/8 | 5/9 | | 5/11 | 5/12 | 5/13 | 5/14 | | 5/16 | 5/17 | 5/18 | 5/19 | | .... |
| 6/1 [17] | | | | 6/5 [37] | | 6/7 | | | | 6/11 | | 6/13 | | | | 6/17 | | 6/19 | | .... |
| 7/1 [18] | 7/2 [26] | 7/3 [29] | 7/4 [38] | 7/5 | 7/6 | | 7/8 | 7/9 | 7/10 | 7/11 | 7/12 | 7/13 | | 7/15 | 7/16 | 7/17 | 7/18 | 7/19 | 7/20 | .... |
| 8/1 [27] | | 8/3 [39] | | 8/5 | | 8/7 | | 8/9 | | 8/11 | | 8/13 | | 8/15 | | 8/17 | | 8/19 | | .... |
| 9/1 [28] | 9/2 [40] | | 9/4 | 9/5 | | 9/7 | 9/8 | | 9/10 | 9/11 | | 9/13 | 9/14 | | 9/16 | 9/17 | | 9/19 | 9/20 | .... |
| 10/1 [41] | | 10/3 | | | | 10/7 | | 10/9 | | 10/11 | | 10/13 | | | | 10/17 | | 10/19 | | .... |
| 11/1 | 11/2 | 11/3 | 11/4 | 11/5 | 11/6 | 11/7 | 11/8 | 11/9 | 11/10 | | 11/12 | 11/13 | 11/14 | 11/15 | 11/16 | 11/17 | 11/18 | 11/19 | 11/20 | .... |
| 12/1 | | | | 12/5 | | 12/7 | | | | 12/11 | | 12/13 | | | | 12/17 | | 12/19 | | .... |
| 13/1 | 13/2 | 13/3 | 13/4 | 13/5 | 13/6 | 13/7 | 13/8 | 13/9 | 13/10 | 13/11 | 13/12 | | 13/14 | 13/15 | 13/16 | 13/17 | 13/18 | 13/19 | 13/20 | .... |
| 14/1 | | 14/3 | | 14/5 | | | | 14/9 | | 14/11 | | 14/13 | | 14/15 | | 14/17 | | 14/19 | | .... |
| 15/1 | 15/2 | | 15/4 | | | 15/7 | 15/8 | | | 15/11 | | 15/13 | 15/14 | | 15/16 | 15/17 | | 15/19 | | .... |
| 16/1 | | 16/3 | | 16/5 | | 16/7 | | 16/9 | | 16/11 | | 16/13 | | 16/15 | | 16/17 | | 16/19 | | .... |
| 17/1 | 17/2 | 17/3 | 17/4 | 17/5 | 17/6 | 17/7 | 17/8 | 17/9 | 17/10 | 17/11 | 17/12 | 17/13 | 17/14 | 17/15 | 17/16 | | 17/18 | 17/19 | 17/20 | .... |
| 18/1 | | | | 18/5 | | 18/7 | | | | 18/11 | | 18/13 | | | | 18/17 | | 18/19 | | .... |
| 19/1 | 19/2 | 19/3 | 19/4 | 19/5 | 19/6 | 19/7 | 19/8 | 19/9 | 19/10 | 19/11 | 19/12 | 19/13 | 19/14 | 19/15 | 19/16 | 19/17 | 19/18 | | 19/20 | .... |
| 20/1 | | 20/3 | | | | 20/7 | | 20/9 | | 20/11 | | 20/13 | | | | 20/17 | | 20/19 | | .... |
| .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... |

Table 3

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | .... |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1/1 [1] | 1/2 [2] | 1/3 [5] | 1/4 [6] | 1/5 [11] | 1/6 [12] | 1/7 [21] | 1/8 [22] | 1/9 [31] | 1/10 [32] | 1/11 [45] | 1/12 [46] | 1/13 [63] | 1/14 [64] | 1/15 [79] | 1/16 [80] | 1/17 [101] | 1/18 [102] | 1/19 [127] | 1/20 [128] | .... |
| 2/1 [3] | | 2/3 [7] | | 2/5 [13] | | 2/7 [23] | | 2/9 [33] | | 2/11 [47] | | 2/13 [65] | | 2/15 [81] | | 2/17 [103] | | 2/19 [129] | | .... |
| 3/1 [4] | 3/2 [8] | | 3/4 [14] | 3/5 [20] | | 3/7 [30] | 3/8 [34] | | 3/10 [48] | 3/11 [62] | | 3/13 [78] | 3/14 [82] | | 3/16 [104] | 3/17 [126] | | 3/19 | 3/20 | .... |
| 4/1 [9] | | 4/3 [15] | | 4/5 [24] | | 4/7 [35] | | 4/9 [49] | | 4/11 [66] | | 4/13 [83] | | 4/15 [105] | | 4/17 [130] | | 4/19 | | .... |
| 5/1 [10] | 5/2 [16] | 5/3 [19] | 5/4 [25] | | 5/6 [36] | 5/7 [44] | 5/8 [50] | 5/9 [61] | | 5/11 [77] | 5/12 [84] | 5/13 [100] | 5/14 [106] | | 5/16 [131] | 5/17 | 5/18 | 5/19 | | .... |
| 6/1 [17] | | | | 6/5 [37] | | 6/7 [51] | | | | 6/11 [85] | | 6/13 [107] | | | | 6/17 | | 6/19 | | .... |
| 7/1 [18] | 7/2 [26] | 7/3 [29] | 7/4 [38] | 7/5 [43] | 7/6 [52] | | 7/8 [67] | 7/9 [76] | 7/10 [86] | 7/11 [99] | 7/12 [108] | 7/13 [125] | | 7/15 | 7/16 | 7/17 | 7/18 | 7/19 | 7/20 | .... |
| 8/1 [27] | | 8/3 [39] | | 8/5 [53] | | 8/7 [68] | | 8/9 [87] | | 8/11 [109] | | 8/13 [132] | | 8/15 | | 8/17 | | 8/19 | | .... |
| 9/1 [28] | 9/2 [40] | | 9/4 [54] | 9/5 [60] | | 9/7 [75] | 9/8 [88] | | 9/10 [110] | 9/11 [124] | | 9/13 | 9/14 | | 9/16 | 9/17 | | 9/19 | 9/20 | .... |
| 10/1 [41] | | 10/3 [55] | | | | 10/7 [89] | | 10/9 [111] | | 10/11 [133] | | 10/13 | | | | 10/17 | | 10/19 | | .... |
| 11/1 [42] | 11/2 [56] | 11/3 [59] | 11/4 [69] | 11/5 [74] | 11/6 [90] | 11/7 [98] | 11/8 [112] | 11/9 [123] | 11/10 [134] | | 11/12 | 11/13 | 11/14 | 11/15 | 11/16 | 11/17 | 11/18 | 11/19 | 11/20 | .... |
| 12/1 [57] | | | | 12/5 [91] | | 12/7 [113] | | | | 12/11 | | 12/13 | | | | 12/17 | | 12/19 | | .... |
| 13/1 [58] | 13/2 [70] | 13/3 [73] | 13/4 [92] | 13/5 [97] | 13/6 [114] | 13/7 [122] | 13/8 [135] | 13/9 | 13/10 | 13/11 | 13/12 | | 13/14 | 13/15 | 13/16 | 13/17 | 13/18 | 13/19 | 13/20 | .... |
| 14/1 [71] | | 14/3 [93] | | 14/5 [115] | | | | 14/9 | | 14/11 | | 14/13 | | 14/15 | | 14/17 | | 14/19 | | .... |
| 15/1 [72] | 15/2 [94] | | 15/4 [116] | | | 15/7 | 15/8 | | | 15/11 | | 15/13 | 15/14 | | 15/16 | 15/17 | | 15/19 | | .... |
| 16/1 [95] | | 16/3 [117] | | 16/5 [136] | | 16/7 | | 16/9 | | 16/11 | | 16/13 | | 16/15 | | 16/17 | | 16/19 | | .... |
| 17/1 [96] | 17/2 [118] | 17/3 [121] | 17/4 [137] | 17/5 | 17/6 | 17/7 | 17/8 | 17/9 | 17/10 | 17/11 | 17/12 | 17/13 | 17/14 | 17/15 | 17/16 | | 17/18 | 17/19 | 17/20 | .... |
| 18/1 [119] | | | | 18/5 | | 18/7 | | | | 18/11 | | 18/13 | | | | 18/17 | | 18/19 | | .... |
| 19/1 [120] | 19/2 [138] | 19/3 | 19/4 | 19/5 | 19/6 | 19/7 | 19/8 | 19/9 | 19/10 | 19/11 | 19/12 | 19/13 | 19/14 | 19/15 | 19/16 | 19/17 | 19/18 | | 19/20 | .... |
| 20/1 [139] | | 20/3 | | | | 20/7 | | 20/9 | | 20/11 | | 20/13 | | | | 20/17 | | 20/19 | | .... |
| .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... | .... |

Table 4

| | Group 1 | | | | | | | | | | Group 2 | | | | | | | | | | Group 3 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | .... |
| | 1/1 | 1/2 | 1/3 | 1/4 | 1/5 | 1/6 | 1/7 | 1/8 | 1/9 | 1/10 | 1/11 | 1/12 | 1/13 | 1/14 | 1/15 | 1/16 | 1/17 | 1/18 | 1/19 | 1/20 | 1/21 | 1/22 | 1/23 | 1/24 | 1/25 | 1/26 | 1/27 | 1/28 | 1/29 | 1/30 | 1/31 | 1/32 | |
| | 2/1 | 2/2 | 2/3 | 2/4 | 2/5 | 2/6 | 2/7 | 2/8 | 2/9 | 2/10 | 2/11 | 2/12 | 2/13 | 2/14 | 2/15 | 2/16 | 2/17 | 2/18 | 2/19 | 2/20 | 2/21 | 2/22 | 2/23 | 2/24 | 2/25 | 2/26 | 2/27 | 2/28 | 2/29 | 2/30 | 2/31 | 2/32 | |
| | 3/1 | 3/2 | 3/3 | 3/4 | 3/5 | 3/6 | 3/7 | 3/8 | 3/9 | 3/10 | 3/11 | 3/12 | 3/13 | 3/14 | 3/15 | 3/16 | 3/17 | 3/18 | 3/19 | 3/20 | 3/21 | 3/22 | 3/23 | 3/24 | 3/25 | 3/26 | 3/27 | 3/28 | 3/29 | 3/30 | 3/31 | 3/32 | |
| | 4/1 | 4/2 | 4/3 | 4/4 | 4/5 | 4/6 | 4/7 | 4/8 | 4/9 | 4/10 | 4/11 | 4/12 | 4/13 | 4/14 | 4/15 | 4/16 | 4/17 | 4/18 | 4/19 | 4/20 | 4/21 | 4/22 | 4/23 | 4/24 | 4/25 | 4/26 | 4/27 | 4/28 | 4/29 | 4/30 | 4/31 | 4/32 | |
| A | 5/1 | 5/2 | 5/3 | 5/4 | 5/5 | 5/6 | 5/7 | 5/8 | 5/9 | 5/10 | 5/11 | 5/12 | 5/13 | 5/14 | 5/15 | 5/16 | 5/17 | 5/18 | 5/19 | 5/20 | 5/21 | 5/22 | 5/23 | 5/24 | 5/25 | 5/26 | 5/27 | 5/28 | 5/29 | 5/30 | 5/31 | 5/32 | |
| | 6/1 | 6/2 | 6/3 | 6/4 | 6/5 | 6/6 | 6/7 | 6/8 | 6/9 | 6/10 | 6/11 | 6/12 | 6/13 | 6/14 | 6/15 | 6/16 | 6/17 | 6/18 | 6/19 | 6/20 | 6/21 | 6/22 | 6/23 | 6/24 | 6/25 | 6/26 | 6/27 | 6/28 | 6/29 | 6/30 | 6/31 | 6/32 | |
| | 7/1 | 7/2 | 7/3 | 7/4 | 7/5 | 7/6 | 7/7 | 7/8 | 7/9 | 7/10 | 7/11 | 7/12 | 7/13 | 7/14 | 7/15 | 7/16 | 7/17 | 7/18 | 7/19 | 7/20 | 7/21 | 7/22 | 7/23 | 7/24 | 7/25 | 7/26 | 7/27 | 7/28 | 7/29 | 7/30 | 7/31 | 7/32 | |
| | 8/1 | 8/2 | 8/3 | 8/4 | 8/5 | 8/6 | 8/7 | 8/8 | 8/9 | 8/10 | 8/11 | 8/12 | 8/13 | 8/14 | 8/15 | 8/16 | 8/17 | 8/18 | 8/19 | 8/20 | 8/21 | 8/22 | 8/23 | 8/24 | 8/25 | 8/26 | 8/27 | 8/28 | 8/29 | 8/30 | 8/31 | 8/32 | |
| | 9/1 | 9/2 | 9/3 | 9/4 | 9/5 | 9/6 | 9/7 | 9/8 | 9/9 | 9/10 | 9/11 | 9/12 | 9/13 | 9/14 | 9/15 | 9/16 | 9/17 | 9/18 | 9/19 | 9/20 | 9/21 | 9/22 | 9/23 | 9/24 | 9/25 | 9/26 | 9/27 | 9/28 | 9/29 | 9/30 | 9/31 | 9/32 | |
| | 10/1 | 10/2 | 10/3 | 10/4 | 10/5 | 10/6 | 10/7 | 10/8 | 10/9 | 10/10 | 10/11 | 10/12 | 10/13 | 10/14 | 10/15 | 10/16 | 10/17 | 10/18 | 10/19 | 10/20 | 10/21 | 10/22 | 10/23 | 10/24 | 10/25 | 10/26 | 10/27 | 10/28 | 10/29 | 10/30 | 10/31 | 10/32 | |
| | 11/1 | 11/2 | 11/3 | 11/4 | 11/5 | 11/6 | 11/7 | 11/8 | 11/9 | 11/10 | 11/11 | 11/12 | 11/13 | 11/14 | 11/15 | 11/16 | 11/17 | 11/18 | 11/19 | 11/20 | 11/21 | 11/22 | 11/23 | 11/24 | 11/25 | 11/26 | 11/27 | 11/28 | 11/29 | 11/30 | 11/31 | 11/32 | |
| | 12/1 | 12/2 | 12/3 | 12/4 | 12/5 | 12/6 | 12/7 | 12/8 | 12/9 | 12/10 | 12/11 | 12/12 | 12/13 | 12/14 | 12/15 | 12/16 | 12/17 | 12/18 | 12/19 | 12/20 | 12/21 | 12/22 | 12/23 | 12/24 | 12/25 | 12/26 | 12/27 | 12/28 | 12/29 | 12/30 | 12/31 | 12/32 | |
| | 13/1 | 13/2 | 13/3 | 13/4 | 13/5 | 13/6 | 13/7 | 13/8 | 13/9 | 13/10 | 13/11 | 13/12 | 13/13 | 13/14 | 13/15 | 13/16 | 13/17 | 13/18 | 13/19 | 13/20 | 13/21 | 13/22 | 13/23 | 13/24 | 13/25 | 13/26 | 13/27 | 13/28 | 13/29 | 13/30 | 13/31 | 13/32 | |
| | 14/1 | 14/2 | 14/3 | 14/4 | 14/5 | 14/6 | 14/7 | 14/8 | 14/9 | 14/10 | 14/11 | 14/12 | 14/13 | 14/14 | 14/15 | 14/16 | 14/17 | 14/18 | 14/19 | 14/20 | 14/21 | 14/22 | 14/23 | 14/24 | 14/25 | 14/26 | 14/27 | 14/28 | 14/29 | 14/30 | 14/31 | 14/32 | |
| B | 15/1 | 15/2 | 15/3 | 15/4 | 15/5 | 15/6 | 15/7 | 15/8 | 15/9 | 15/10 | 15/11 | 15/12 | 15/13 | 15/14 | 15/15 | 15/16 | 15/17 | 15/18 | 15/19 | 15/20 | 15/21 | 15/22 | 15/23 | 15/24 | 15/25 | 15/26 | 15/27 | 15/28 | 15/29 | 15/30 | 15/31 | 15/32 | |
| | 16/1 | 16/2 | 16/3 | 16/4 | 16/5 | 16/6 | 16/7 | 16/8 | 16/9 | 16/10 | 16/11 | 16/12 | 16/13 | 16/14 | 16/15 | 16/16 | 16/17 | 16/18 | 16/19 | 16/20 | 16/21 | 16/22 | 16/23 | 16/24 | 16/25 | 16/26 | 16/27 | 16/28 | 16/29 | 16/30 | 16/31 | 16/32 | |
| | 17/1 | 17/2 | 17/3 | 17/4 | 17/5 | 17/6 | 17/7 | 17/8 | 17/9 | 17/10 | 17/11 | 17/12 | 17/13 | 17/14 | 17/15 | 17/16 | 17/17 | 17/18 | 17/19 | 17/20 | 17/21 | 17/22 | 17/23 | 17/24 | 17/25 | 17/26 | 17/27 | 17/28 | 17/29 | 17/30 | 17/31 | 17/32 | |
| | 18/1 | 18/2 | 18/3 | 18/4 | 18/5 | 18/6 | 18/7 | 18/8 | 18/9 | 18/10 | 18/11 | 18/12 | 18/13 | 18/14 | 18/15 | 18/16 | 18/17 | 18/18 | 18/19 | 18/20 | 18/21 | 18/22 | 18/23 | 18/24 | 18/25 | 18/26 | 18/27 | 18/28 | 18/29 | 18/30 | 18/31 | 18/32 | |
| | 19/1 | 19/2 | 19/3 | 19/4 | 19/5 | 19/6 | 19/7 | 19/8 | 19/9 | 19/10 | 19/11 | 19/12 | 19/13 | 19/14 | 19/15 | 19/16 | 19/17 | 19/18 | 19/19 | 19/20 | 19/21 | 19/22 | 19/23 | 19/24 | 19/25 | 19/26 | 19/27 | 19/28 | 19/29 | 19/30 | 19/31 | 19/32 | |
| | 20/1 | 20/2 | 20/3 | 20/4 | 20/5 | 20/6 | 20/7 | 20/8 | 20/9 | 20/10 | 20/11 | 20/12 | 20/13 | 20/14 | 20/15 | 20/16 | 20/17 | 20/18 | 20/19 | 20/20 | 20/21 | 20/22 | 20/23 | 20/24 | 20/25 | 20/26 | 20/27 | 20/28 | 20/29 | 20/30 | 20/31 | 20/32 | |
| | 21/1 | 21/2 | 21/3 | 21/4 | 21/5 | 21/6 | 21/7 | 21/8 | 21/9 | 21/10 | 21/11 | 21/12 | 21/13 | 21/14 | 21/15 | 21/16 | 21/17 | 21/18 | 21/19 | 21/20 | 21/21 | 21/22 | 21/23 | 21/24 | 21/25 | 21/26 | 21/27 | 21/28 | 21/29 | 21/30 | 21/31 | 21/32 | |
| | 22/1 | 22/2 | 22/3 | 22/4 | 22/5 | 22/6 | 22/7 | 22/8 | 22/9 | 22/10 | 22/11 | 22/12 | 22/13 | 22/14 | 22/15 | 22/16 | 22/17 | 22/18 | 22/19 | 22/20 | 22/21 | 22/22 | 22/23 | 22/24 | 22/25 | 22/26 | 22/27 | 22/28 | 22/29 | 22/30 | 22/31 | 22/32 | |
| | 23/1 | 23/2 | 23/3 | 23/4 | 23/5 | 23/6 | 23/7 | 23/8 | 23/9 | 23/10 | 23/11 | 23/12 | 23/13 | 23/14 | 23/15 | 23/16 | 23/17 | 23/18 | 23/19 | 23/20 | 23/21 | 23/22 | 23/23 | 23/24 | 23/25 | 23/26 | 23/27 | 23/28 | 23/29 | 23/30 | 23/31 | 23/32 | |
| | 24/1 | 24/2 | 24/3 | 24/4 | 24/5 | 24/6 | 24/7 | 24/8 | 24/9 | 24/10 | 24/11 | 24/12 | 24/13 | 24/14 | 24/15 | 24/16 | 24/17 | 24/18 | 24/19 | 24/20 | 24/21 | 24/22 | 24/23 | 24/24 | 24/25 | 24/26 | 24/27 | 24/28 | 24/29 | 24/30 | 24/31 | 24/32 | |
| C | 25/1 | 25/2 | 25/3 | 25/4 | 25/5 | 25/6 | 25/7 | 25/8 | 25/9 | 25/10 | 25/11 | 25/12 | 25/13 | 25/14 | 25/15 | 25/16 | 25/17 | 25/18 | 25/19 | 25/20 | 25/21 | 25/22 | 25/23 | 25/24 | 25/25 | 25/26 | 25/27 | 25/28 | 25/29 | 25/30 | 25/31 | 25/32 | |
| | 26/1 | 26/2 | 26/3 | 26/4 | 26/5 | 26/6 | 26/7 | 26/8 | 26/9 | 26/10 | 26/11 | 26/12 | 26/13 | 26/14 | 26/15 | 26/16 | 26/17 | 26/18 | 26/19 | 26/20 | 26/21 | 26/22 | 26/23 | 26/24 | 26/25 | 26/26 | 26/27 | 26/28 | 26/29 | 26/30 | 26/31 | 26/32 | |
| | 27/1 | 27/2 | 27/3 | 27/4 | 27/5 | 27/6 | 27/7 | 27/8 | 27/9 | 27/10 | 27/11 | 27/12 | 27/13 | 27/14 | 27/15 | 27/16 | 27/17 | 27/18 | 27/19 | 27/20 | 27/21 | 27/22 | 27/23 | 27/24 | 27/25 | 27/26 | 27/27 | 27/28 | 27/29 | 27/30 | 27/31 | 27/32 | |
| | 28/1 | 28/2 | 28/3 | 28/4 | 28/5 | 28/6 | 28/7 | 28/8 | 28/9 | 28/10 | 28/11 | 28/12 | 28/13 | 28/14 | 28/15 | 28/16 | 28/17 | 28/18 | 28/19 | 28/20 | 28/21 | 28/22 | 28/23 | 28/24 | 28/25 | 28/26 | 28/27 | 28/28 | 28/29 | 28/30 | 28/31 | 28/32 | |
| | 29/1 | 29/2 | 29/3 | 29/4 | 29/5 | 29/6 | 29/7 | 29/8 | 29/9 | 29/10 | 29/11 | 29/12 | 29/13 | 29/14 | 29/15 | 29/16 | 29/17 | 29/18 | 29/19 | 29/20 | 29/21 | 29/22 | 29/23 | 29/24 | 29/25 | 29/26 | 29/27 | 29/28 | 29/29 | 29/30 | 29/31 | 29/32 | |
| | 30/1 | 30/2 | 30/3 | 30/4 | 30/5 | 30/6 | 30/7 | 30/8 | 30/9 | 30/10 | 30/11 | 30/12 | 30/13 | 30/14 | 30/15 | 30/16 | 30/17 | 30/18 | 30/19 | 30/20 | 30/21 | 30/22 | 30/23 | 30/24 | 30/25 | 30/26 | 30/27 | 30/28 | 30/29 | 30/30 | 30/31 | 30/32 | |
| | 31/1 | 31/2 | 31/3 | 31/4 | 31/5 | 31/6 | 31/7 | 31/8 | 31/9 | 31/10 | 31/11 | 31/12 | 31/13 | 31/14 | 31/15 | 31/16 | 31/17 | 31/18 | 31/19 | 31/20 | 31/21 | 31/22 | 31/23 | 31/24 | 31/25 | 31/26 | 31/27 | 31/28 | 31/29 | 31/30 | 31/31 | 31/32 | |
| | 32/1 | 32/2 | 32/3 | 32/4 | 32/5 | 32/6 | 32/7 | 32/8 | 32/9 | 32/10 | 32/11 | 32/12 | 32/13 | 32/14 | 32/15 | 32/16 | 32/17 | 32/18 | 32/19 | 32/20 | 32/21 | 32/22 | 32/23 | 32/24 | 32/25 | 32/26 | 32/27 | 32/28 | 32/29 | 32/30 | 32/31 | 32/32 | |

Table 5

# Group 93

| 921 | 922 | 923 | 924 | 925 | 926 | 927 | 928 | 929 | 930 |
|---|---|---|---|---|---|---|---|---|---|
| 1/921 | 1/922 | 1/923 | 1/924 | 1/925 | 1/926 | 1/927 | 1/928 | 1/929 | 1/930 |
| 2/921 | 2/922 | 2/923 | 2/924 | 2/925 | 2/926 | 2/927 | 2/928 | 2/929 | 2/930 |
| 3/921 | 3/922 | 3/923 | 3/924 | 3/925 | 3/926 | 3/927 | 3/928 | 3/929 | 3/930 |
| 4/921 | 4/922 | 4/923 | 4/924 | 4/925 | 4/926 | 4/927 | 4/928 | 4/929 | 4/930 |
| 5/921 | 5/922 | 5/923 | 5/924 | 5/925 | 5/926 | 5/927 | 5/928 | 5/929 | 5/930 |
| 6/921 | 6/922 | 6/923 | 6/924 | 6/925 | 6/926 | 6/927 | 6/928 | 6/929 | 6/930 |
| 7/921 | 7/922 | 7/923 | 7/924 | 7/925 | 7/926 | 7/927 | 7/928 | 7/929 | 7/930 |
| 8/921 | 8/922 | 8/923 | 8/924 | 8/925 | 8/926 | 8/927 | 8/928 | 8/929 | 8/930 |
| 9/921 | 9/922 | 9/923 | 9/924 | 9/925 | 9/926 | 9/927 | 9/928 | 9/929 | 9/930 |
| 10/921 | 10/922 | 10/923 | 10/924 | 10/925 | 10/926 | 10/927 | 10/928 | 10/929 | 10/930 |
| 11/921 | 11/922 | 11/923 | 11/924 | 11/925 | 11/926 | 11/927 | 11/928 | 11/929 | 11/930 |
| 12/921 | 12/922 | 12/923 | 12/924 | 12/925 | 12/926 | 12/927 | 12/928 | 12/929 | 12/930 |
| 13/921 | 13/922 | 13/923 | 13/924 | 13/925 | 13/926 | 13/927 | 13/928 | 13/929 | 13/930 |
| 14/921 | 14/922 | 14/923 | 14/924 | 14/925 | 14/926 | 14/927 | 14/928 | 14/929 | 14/930 |
| 15/921 | 15/922 | 15/923 | 15/924 | 15/925 | 15/926 | 15/927 | 15/928 | 15/929 | 15/930 |
| 16/921 | 16/922 | 16/923 | 16/924 | 16/925 | 16/926 | 16/927 | 16/928 | 16/929 | 16/930 |
| 17/921 | 17/922 | 17/923 | 17/924 | 17/925 | 17/926 | 17/927 | 17/928 | 17/929 | 17/930 |
| 18/921 | 18/922 | 18/923 | 18/924 | 18/925 | 18/926 | 18/927 | 18/928 | 18/929 | 18/930 |
| 19/921 | 19/922 | 19/923 | 19/924 | 19/925 | 19/926 | 19/927 | 19/928 | 19/929 | 19/930 |
| 20/921 | 20/922 | 20/923 | 20/924 | 20/925 | 20/926 | 20/927 | 20/928 | 20/929 | 20/930 |
| .... | .... | .... | .... | .... | .... | .... | .... | .... | .... |

In the part of the infinite set

of positive rational numbers

presented in this table,

there are 18,500 rational numbers

(including repeating ones).

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 921/1 | 921/2 | 921/3 | 921/4 | 921/5 | 921/6 | 921/7 | 921/8 | 921/9 | 921/10 | 921/11 | 921/12 | 921/13 | 921/14 | 921/15 | 921/16 | 921/17 | 921/18 | 921/19 | 921/20 | .... | 921/921 | 921/922 | 921/923 | 921/924 | 921/925 | 921/926 | 921/927 | 921/928 | 921/929 | 921/930 |
| 922/1 | 922/2 | 922/3 | 922/4 | 922/5 | 922/6 | 922/7 | 922/8 | 922/9 | 922/10 | 922/11 | 922/12 | 922/13 | 922/14 | 922/15 | 922/16 | 922/17 | 922/18 | 922/19 | 922/20 | .... | 922/921 | 922/922 | 922/923 | 922/924 | 922/925 | 922/926 | 922/927 | 922/928 | 922/929 | 922/930 |
| 923/1 | 923/2 | 923/3 | 923/4 | 923/5 | 923/6 | 923/7 | 923/8 | 923/9 | 923/10 | 923/11 | 923/12 | 923/13 | 923/14 | 923/15 | 923/16 | 923/17 | 923/18 | 923/19 | 923/20 | .... | 923/921 | 923/922 | 923/923 | 923/924 | 923/925 | 923/926 | 923/927 | 923/928 | 923/929 | 923/930 |
| 924/1 | 924/2 | 924/3 | 924/4 | 924/5 | 924/6 | 924/7 | 924/8 | 924/9 | 924/10 | 924/11 | 924/12 | 924/13 | 924/14 | 924/15 | 924/16 | 924/17 | 924/18 | 924/19 | 924/20 | .... | 924/921 | 924/922 | 924/923 | 924/924 | 924/925 | 924/926 | 924/927 | 924/928 | 924/929 | 924/930 |
| 925/1 | 925/2 | 925/3 | 925/4 | 925/5 | 925/6 | 925/7 | 925/8 | 925/9 | 925/10 | 925/11 | 925/12 | 925/13 | 925/14 | 925/15 | 925/16 | 925/17 | 925/18 | 925/19 | 925/20 | .... | 925/921 | 925/922 | 925/923 | 925/924 | 925/925 | 925/926 | 925/927 | 925/928 | 925/929 | 925/930 |
| 926/1 | 926/2 | 926/3 | 926/4 | 926/5 | 926/6 | 926/7 | 926/8 | 926/9 | 926/10 | 926/11 | 926/12 | 926/13 | 926/14 | 926/15 | 926/16 | 926/17 | 926/18 | 926/19 | 926/20 | .... | 926/921 | 926/922 | 926/923 | 926/924 | 926/925 | 926/926 | 926/927 | 926/928 | 926/929 | 926/930 |
| 927/1 | 927/2 | 927/3 | 927/4 | 927/5 | 927/6 | 927/7 | 927/8 | 927/9 | 927/10 | 927/11 | 927/12 | 927/13 | 927/14 | 927/15 | 927/16 | 927/17 | 927/18 | 927/19 | 927/20 | .... | 927/921 | 927/922 | 927/923 | 927/924 | 927/925 | 927/926 | 927/927 | 927/928 | 927/929 | 927/930 |
| 928/1 | 928/2 | 928/3 | 928/4 | 928/5 | 928/6 | 928/7 | 928/8 | 928/9 | 928/10 | 928/11 | 928/12 | 928/13 | 928/14 | 928/15 | 928/16 | 928/17 | 928/18 | 928/19 | 928/20 | .... | 928/921 | 928/922 | 928/923 | 928/924 | 928/925 | 928/926 | 928/927 | 928/928 | 928/929 | 928/930 |
| 929/1 | 929/2 | 929/3 | 929/4 | 929/5 | 929/6 | 929/7 | 929/8 | 929/9 | 929/10 | 929/11 | 929/12 | 929/13 | 929/14 | 929/15 | 929/16 | 929/17 | 929/18 | 929/19 | 929/20 | .... | 929/921 | 929/922 | 929/923 | 929/924 | 929/925 | 929/926 | 929/927 | 929/928 | 929/929 | 929/930 |
| 930/1 | 930/2 | 930/3 | 930/4 | 930/5 | 930/6 | 930/7 | 930/8 | 930/9 | 930/10 | 930/11 | 930/12 | 930/13 | 930/14 | 930/15 | 930/16 | 930/17 | 930/18 | 930/19 | 930/20 | .... | 930/921 | 930/922 | 930/923 | 930/924 | 930/925 | 930/926 | 930/927 | 930/928 | 930/929 | 930/930 |

Table 6